

Appendix (Monetary Transfers in the U.S.: How Efficient are Tax Rebates?)

A. System of Equations in real terms

$$(A1) \quad \pi_t = \frac{s_t}{s_{t-1}} \pi_t^*$$

$$(A2) \quad \pi_t C_t = m_t^c + (1 - \varphi)(\theta_t - 1)m_t$$

$$(A3) \quad \Lambda_t w_t = - \frac{\chi(1 - H_t - \Omega_t)^{-\gamma^{-1}}}{\left(C_t - \frac{\chi(1 - H_t - \Omega_t)^{1-\gamma^{-1}}}{1 - \gamma^{-1}} \right)^{\sigma^{-1}}}$$

$$(A4) \quad \Lambda_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\pi_{t+1}} (1 + i_{t+1}) \right]$$

$$(A5) \quad \Lambda_t = \beta E_t \left[(1 + i_{t+1}^*) \frac{s_{t+1}}{s_t} \frac{\Lambda_{t+1}}{\pi_{t+1}} \right]$$

$$(A6) \quad \Lambda_t w_t \frac{\xi}{m_t^c} \pi_t (\Delta M_t^c - \theta) + \Lambda_t = \beta E_t \left[\frac{1}{\pi_{t+1} \left(C_{t+1} - \frac{\chi(1 - H_{t+1} - \Omega_{t+1})^{1-\gamma^{-1}}}{1 - \gamma^{-1}} \right)^{\sigma^{-1}}} \right. \\ \left. + \Lambda_{t+1} w_{t+1} \frac{\xi}{\frac{m_{t+1}^c}{1}} \Delta M_{t+1}^c (\Delta M_{t+1}^c - \theta) \right]$$

$$(A7) \quad Y_t = e^{z_t} K_t^\alpha H_t^{1-\alpha}$$

$$(A8) \quad I_t = K_{t+1} - (1 - \delta)K_t$$

$$(A9) \quad w_t = (1 - \alpha) \frac{Y_t}{H_t}$$

$$(A10) \quad (1 + i_t) + \psi(K_{t+1} - K_t) = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) + \psi(K_{t+2} - K_{t+1}) \right) \right]$$

$$(A11) \quad m_{t+1} = \theta_t \frac{m_t}{\pi_t}$$

$$(A12) \quad \pi_t I_t = m_t^b + \varphi(\theta_t - 1)m_t$$

$$(A13) \quad b_{t+1} - \frac{s_t}{s_{t-1}}(1 + i_t^*) \frac{b_t}{\pi_t} = Y_t - c_t - I_t - \frac{\psi}{2}(K_{t+1} - K_t)^2$$

To avoid clutter we define changes in money cash balances as

$$(A14) \quad \Delta M_t^c = \frac{m_{t+1}^c \pi_t}{m_t^c}$$

By definition on money balances we have that

$$(A15) \quad m_t = m_t^b + m_t^c$$

$$(A16) \quad i_t^* = i^W - \tau b_t$$

$$(A17) \quad \log(\theta_{t+1}) = (1 - \rho_\theta) \log(\bar{\theta}) + \rho_\theta \log(\theta_t) + \varepsilon_{\theta t+1}$$

$$(A18) \quad \log(z_{t+1}) = (1 - \rho_z) \log(\bar{z}) + \rho_z \log(z_t) + \varepsilon_{zt+1}$$

In the Case of the CES utility function, equations A3 and A6 are replaced by

$$(A3') \quad \Lambda_t w_t = \frac{\gamma C_t^{(1-\sigma)(1-\gamma)}}{(1 - H_t - \Omega_t)^{1-\gamma+\sigma\gamma}}$$

$$(A6') \quad \Lambda_t w_t \frac{\xi}{m_t^c} \pi_t (\Delta M_t^c - \theta) + \Lambda_t =$$

$$\beta E_t \left[\frac{(1-\gamma)(1-H_{t+1}-\Omega_{t+1})^{\gamma(1-\sigma)}}{\pi_{t+1} C_{t+1}^{\gamma+\sigma(1-\gamma)}} + \Lambda_{t+1} w_{t+1} \frac{\xi}{m_{t+1}^c} \Delta M_{t+1}^c (\Delta M_{t+1}^c - \theta) \right]$$

B. Steady State Derivations

Adjustment costs disappear in the steady state, and we assume that in the long-run equilibrium the domestic gross inflation rate is given by the gross money growth rate so that $\pi = \theta$.

From equation (A3) and, after some manipulation, we have that the domestic nominal interest rate in steady state is

$$i = \frac{\pi}{\beta} - 1$$

We look at a steady state in which the domestic and foreign inflation levels are the same, so purchasing power parity implies that the change in the nominal exchange rate is constant. Consequently the uncovered interest rate parity condition – combining equations (A3) and A5) implies that the domestic and the foreign interest rates are equal ($i = i^*$).

From the definition of the change in money balances (equation (A15) we get that

$$\Delta M^c = \pi$$

To find the steady state capital/output ratio (denoted κ) we get, from the stationarity of equation (A10):

$$\begin{aligned} 1 + i &= \beta \left[\alpha \frac{Y}{K} + (1 - \delta)(1 + i) \right] \\ \frac{1 + i}{\beta} - (1 - \delta)(1 + i) &= \alpha \frac{Y}{K} \\ \kappa \equiv \frac{K}{Y} &= \left[\frac{\alpha \beta}{1 + i - (1 - \delta)(1 + i) \beta} \right] \end{aligned}$$

Then from the production function we can solve for the output

$$Y = \kappa^{\frac{\alpha}{1-\alpha}} H$$

which can be used in equation (A9) to solve for the real wage

$$w = (1 - \alpha) \frac{Y}{H}$$

The steady state physical capital stock will be given by $K = \kappa Y$, and steady state investment by $I = \delta K$.

Since $TB = Y - C - I$ is the trade balance, we can divide by Y both sides of equation (A14) and use the calibration for $\nu = TB/Y$ to obtain the long-run real debt-to-GDP ratio, which is equal to the domestic trade balance as a share of GDP

$$\frac{b}{Y} \left(1 - \frac{1 + i^*}{\pi} \right) = \frac{TB}{Y} = \nu$$

Such that

$$b = \frac{\nu Y}{\left(1 - \frac{1 + i^*}{\pi} \right)}$$

Consequently, the steady state consumption level is given by:

$$C = Y - I - \left(1 - \frac{1 + i^*}{\pi} \right) b$$

Solving for m^c in A2 and using A15 to eliminate m , we get

$$m^c = \frac{\pi C}{(1 + (1 - \varphi)(\theta - 1))} - \frac{(1 - \varphi)(\theta - 1)}{(1 + (1 - \varphi)(\theta - 1))} m^b$$

Also, from A12 we can solve for m^b in terms of m^c

$$m^b = \frac{\Pi \Pi}{1 + \varphi(\theta - 1)} - \frac{\varphi(\theta - 1)}{1 + \varphi(\theta - 1)} m^c$$

Such that if we substitute this m^b in the equation for m^c

$$m^c = \frac{(1 + \varphi(\theta - 1))}{\theta} \pi C - \frac{(1 - \varphi)(\theta - 1)}{\theta} \pi I$$

Given the definition of real money balances, then its steady state level is:

$$m = m^b + m^c$$

The parameter χ is given by

$$\chi = -\frac{w\beta(1-H)^{\gamma^{-1}}}{\pi}$$

From the definition of preferences, and denoting the shadow price associated with household real wealth by $\Lambda_t = P_t \lambda_t$, then the marginal utility of wealth in the steady state is

$$\Lambda = \frac{\beta}{\pi \left(C - \frac{\chi(1-H)^{1-\gamma^{-1}}}{1-\gamma^{-1}} \right) \sigma^{-1}}$$

C. The log-linearized system of equations is given by

$$(C1) \quad 0 = -\hat{\pi}_t + \hat{s}_t - \hat{s}_{t-1}$$

$$(C2) \quad 0 = \hat{\pi}_t + \hat{C}_t - \frac{m^c}{\pi C} \hat{m}_t^c - \frac{(1-\varphi)(\theta-1)m}{\pi C} \hat{m}_t - \frac{(1-\varphi)m}{C} \hat{\theta}_t$$

$$(C3) \quad 0 = \hat{w}_t + \hat{\Lambda}_t + \left(\frac{C}{\sigma \left(C - \frac{\chi(1-H)^{1-\gamma^{-1}}}{1-\gamma^{-1}} \right)} \right) \hat{C}_t - \left(\frac{H}{\gamma(1-H)} - \frac{\chi H(1-H)^{-\gamma^{-1}}}{\sigma \left(C - \frac{\chi(1-H)^{1-\gamma^{-1}}}{1-\gamma^{-1}} \right)} \right) \hat{H}_t$$

$$(C4) \quad 0 = -(m)\hat{m}_t + (m^b)\hat{m}_t^b + (m^c)\hat{m}_t^c$$

$$(C5) \quad 0 = -\hat{w}_t + \hat{Y}_t - \hat{H}_t$$

$$(C6) \quad 0 = -\hat{Y}_t + \alpha \hat{K}_t + (1-\alpha)\hat{H}_t + \hat{z}_t$$

$$(C7) \quad 0 = -\hat{\pi}_t - \hat{I}_t + \frac{m^b}{I\pi} \hat{m}_t^b + \frac{\varphi m(\theta-1)}{I\pi} \hat{m}_t + \frac{\varphi m}{I} \hat{\theta}_t$$

$$(C8) \quad 0 = -\hat{\Delta M}_t^c + \hat{m}_{t+1}^c + \hat{\pi}_t - \hat{m}_t^c$$

$$(C9) \quad 0 = -\hat{b}_{t+1} + \frac{(1+i^*)}{\pi} \hat{s}_t - \frac{(1+i^*)}{\pi} \hat{s}_{t-1} + \frac{(1+i^*)}{\pi} \hat{b}_t - \frac{(1+i^*)}{\pi} \hat{\pi}_t + \frac{i^*}{\pi} \hat{i}_t^* + \frac{Y}{b} \hat{Y}_t - \frac{C}{b} \hat{C}_t - \frac{I}{b} \hat{I}_t$$

$$(C10) \quad 0 = \mathbf{E}_t \left[-\hat{\Lambda}_t + \hat{\Lambda}_{t+1} + \hat{s}_{t+1} - \hat{s}_t - \hat{\pi}_{t+1} + \frac{i^*}{1+i^*} \hat{i}_{t+1}^* \right]$$

$$(C11) \quad 0 = \mathbf{E}_t \left[-\hat{\Lambda}_t + \frac{i}{1+i} \hat{i}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1} \right]$$

$$(C12) \quad 0 = E_t \left[-\Lambda \hat{\Lambda}_t + \beta w \Lambda \frac{\xi}{m^c} (\Delta M^c)^2 \Delta \hat{M}_{t+1}^c - \pi w \Lambda \frac{\xi}{m^c} \Delta M^c \Delta \hat{M}_t \right. \\ \left. - \Lambda \hat{\pi}_{t+1} - \frac{\Lambda C}{\sigma \left(C - \frac{\chi(1-H)^{1-\gamma^{-1}}}{1-\gamma^{-1}} \right)} \hat{C}_{t+1} - \frac{\chi \Lambda H (1-H)^{-\gamma^{-1}}}{\sigma \left(C - \frac{\chi(1-H)^{1-\gamma^{-1}}}{1-\gamma^{-1}} \right)} \hat{H}_{t+1} \right]$$

$$(C13) \quad 0 = E_t \left[\beta \nu K \hat{K}_{t+2} - \left(\nu K (1 + \beta) + \alpha \beta \frac{Y}{K} \right) \hat{K}_{t+1} + \nu K \hat{K}_t - \hat{i}_t + \beta (1 - \delta) \hat{i}_{t+1} + \frac{\alpha \beta Y}{K} \hat{Y}_{t+1} \right. \\ \left. + \left(\frac{\alpha \beta Y}{K} + \beta (1 - \delta) (1 + i) \right) \hat{\Lambda}_{t+1} - \left(\frac{\alpha \beta Y}{K} + \beta (1 - \delta) (1 + i) \right) \hat{\Lambda}_t \right]$$

$$(C14) \quad 0 = \hat{i}_t^* + \frac{\pi b}{i^*} \hat{b}_t$$

$$(C15) \quad 0 = -\hat{m}_{t+1} + \hat{m}_t - \hat{\pi}_t + \hat{\theta}_t$$

$$(C16) \quad 0 = \frac{I}{K} \hat{I}_t - \hat{K}_{t+1} + (1 - \delta) \hat{K}_t$$

$$(C17) \quad \hat{\theta}_{t+1} = \rho_\theta \hat{\theta}_t + \varepsilon_{\theta t+1}$$

$$(C18) \quad \hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_{z t+1}$$

For the CES case

$$(C3') \quad 0 = -\hat{w}_t - \hat{\Lambda}_t + (1 - \gamma)(1 - \sigma) \hat{C}_t + \frac{(1 - \gamma + \gamma \sigma) H}{1 - H} \hat{H}_t$$

$$(C12') \quad 0 = E_t \left[-\Lambda \hat{\Lambda}_t + \beta w \Lambda \frac{\xi}{m^c} (\Delta M^c)^2 \Delta \hat{M}_{t+1}^c - \pi w \Lambda \frac{\xi}{m^c} \Delta M^c \Delta \hat{M}_t \right. \\ \left. - \frac{\beta (1 - \gamma) (1 - H) \Lambda w}{\gamma \pi C} \hat{\pi}_{t+1} - \frac{(\gamma + \sigma (1 - \gamma)) \beta (1 - \gamma) (1 - H) \Lambda w}{\gamma \pi C} \hat{C}_{t+1} - \frac{\gamma (1 - \sigma) H \beta (1 - \gamma) \Lambda w}{\gamma \pi C} \hat{H}_{t+1} \right]$$

D. Technology Shocks

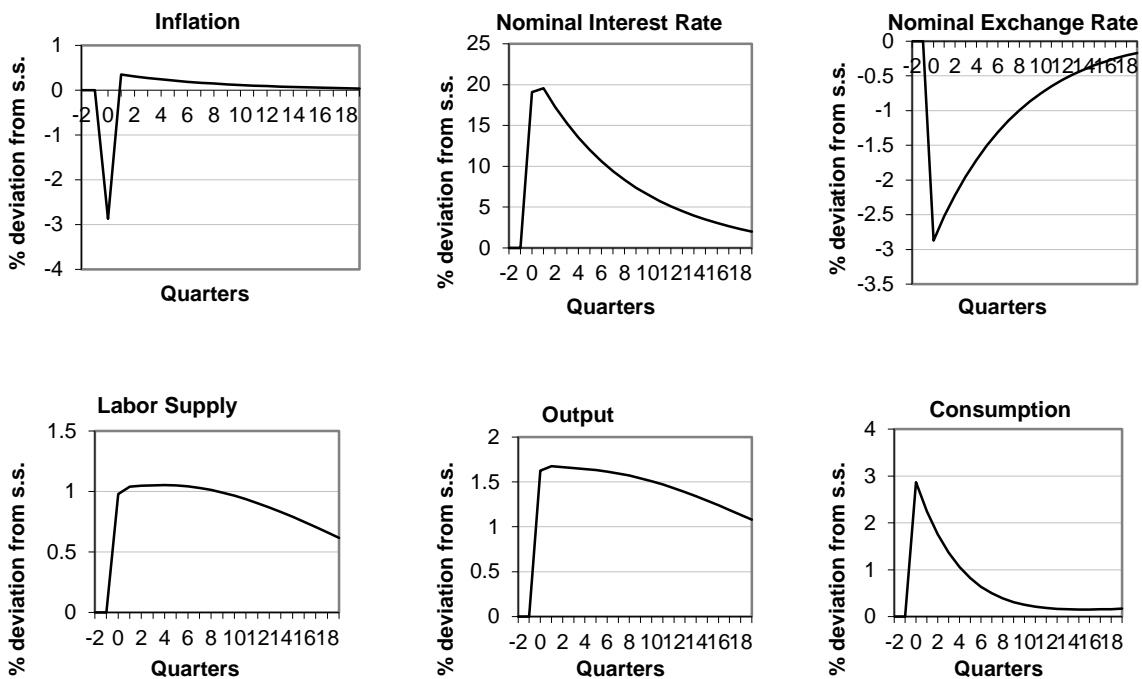


Figure D1: Dynamic response to a 1% technological shock

E. Data

All data comes from the Federal Reserve Bank of St. Louis's FRED data base, covering 1990:q1 to 2008:q3. The particular definition of the variables is the following:

M1 = M1 Money Stock (M1SL), Seasonally Adjusted, in Billions of dollars, quarterly as of end of period.

Real GDP = Real Gross Domestic Product, 1 Decimal (GDPC1), Billions of Chained 2005 Dollars, Seasonally Adjusted Annual Rate, Quarterly.

Real Consumption = Real Personal Consumption Expenditures (PCECC96), Billions of Chained 2005 Dollars, Seasonally Adjusted Annual Rate, Quarterly.

Interest Rate = Interest Rates, Government Securities, Treasury Bills, Percent per Annum, quarterly.

Exchange Rate = National Currency per SDR, period average, quarterly (Source: International Financial Statistics).

Trade Balance = Net Exports of Goods and Services (NETEXP) divided by Gross Domestic Product, 1 Decimal (GDP), both at Seasonally Adjusted Annual Rate, quarterly.

Real Investment = Real Gross Private Domestic Investment, 1 Decimal (GPDIC1), Billions of Chained 2005 Dollars, Seasonally Adjusted Annual Rate, Quarterly.