Appendix for "Remittances, Monetary Policy, and Partial Sterilization", in *Southern Economic Journal*

I. System of Equations

(A1) $\pi_t = \dfrac{s_t}{s_{t-1}}\pi^*$

(A2) $\pi_t C_t = m_t^c + \Gamma_t$

(A3) $\dfrac{\gamma}{1-H_t-\Omega_t} = w_t \Lambda_t$

(A4) $Y_t = e^{z_t} K_t^{\alpha} H_t^{1-\alpha}$

(A5) $I_t = K_{t+1} - (1-\delta)K_t$

(A6) $w_t = (1-\alpha)\dfrac{Y_t}{H_t}$

(A7) $b_{t+1} - \dfrac{s_t}{s_{t-1}}(1+i_t^*)\dfrac{b_t}{\pi_t} = Y_t - C_t - I_t - \dfrac{\upsilon}{2}(K_{t+1}-K_t)^2 + \dfrac{\Gamma_t}{\pi_t}$

Define $\Delta M_t^c \equiv \dfrac{M_{t+1}^c}{M_t^c}$, and by previous the definition

(A8) $\Delta M_t^c = \dfrac{m_{t+1}^c \pi_t}{m_t^c}$

(A9) $m_{t+1} = m_{t+1}^b + m_{t+1}^c$

(A10) $m_{t+1} = \theta_t \dfrac{m_t}{\pi_t}$

(A11) $\pi_t I_t = m_t^b + (\theta_t - 1)m_t$

(A12) $(1+i_t)+\upsilon(K_{t+1}-K_t)=\beta \underset{t}{E}\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left\{\alpha\frac{Y_{t+1}}{K_{t+1}}+(1-\delta)(1+i_{t+1})+\upsilon(K_{t+2}-K_{t+1})\right\}\right]$

(A13) $\Lambda_t=\beta \underset{t}{E}\left[\frac{\Lambda_{t+1}}{\pi_{t+1}}(1+i_{t+1})\right]$

(A14) $\Lambda_t=\beta \underset{t}{E}\left[(1+i^*_{t+1})\frac{s_{t+1}}{s_t}\frac{\Lambda_{t+1}}{\pi_{t+1}}\right]$

(A15) $w_t\Lambda_t\xi\frac{\pi_t}{m^c_t}\left(\Delta M^c_t-\theta\right)+\Lambda_t=\beta \underset{t}{E}\left[\frac{1}{C_{t+1}\pi_{t+1}}\right]+\beta \underset{t}{E}\left[w_{t+1}\Lambda_{t+1}\xi\frac{\Delta M^c_{t+1}}{m^c_{t+1}}\left(\Delta M^c_{t+1}-\theta\right)\right]$

(A16) $\Gamma_t=\underset{t}{E}\left[\vartheta(Y^{ss})^{\tau}\pi_t s_t Y_t^{-\tau}e^{g_t}\right]$

(A17) $i^*_t=i^W-\varphi b_t$

(A18) $\log(\theta_{t+1})=(1-\rho_\theta)\log(\overline{\theta})+\rho_\theta\log(\theta_t)+\varepsilon_{\theta t+1}$

(A19) $\log(g_{t+1})=(1-\rho_g)\log(\overline{g})+\rho_g\log(g_t)+\varepsilon_{gt+1}$

(A20) $\log(z_{t+1})=(1-\rho_z)\log(\overline{z})+\rho_z\log(z_t)+\varepsilon_{zt+1}$

II. Steady States

In the long-run equilibrium we assume the domestic gross inflation rate is given by the gross money growth rate so that $\Pi=\theta$. From (A13) we have that the domestic nominal interest rate in steady state is

$$i=\frac{\Pi}{\beta}-1$$

We can derive the steady state level of remittances from equation (A16) as

$$\Gamma=\vartheta\Pi s$$

To find the steady state capital/output ratio (denoted κ) we use equation (A12) to get:

$$\kappa \equiv \frac{K}{Y} = \left[\frac{\alpha\beta}{1+i-(1-\delta)(1+i)\beta} \right]$$

Then from the production function we can solve for the output/labor ratio

$$\frac{Y}{H} = \kappa^{\frac{\alpha}{1-\alpha}}$$

such that $Y = H\kappa^{\frac{\alpha}{1-\alpha}}$, $K = \frac{Y^{1/\alpha}}{H^{(1-\alpha)/\alpha}}$, and $I = \delta K$.

From equation (A6) we solve for the real wage

$$w = (1-\alpha)\frac{Y}{H}$$

Setting equation (A13) equal to equation (A14) we get that $i = i^*$.

The adjusted trade balance is given by $TB = Y - C - I + \frac{\Gamma}{\Pi}$. Using the calibration for $v = TB/Y$, we obtain the long-run real debt-to-GDP ratio that is equal to the domestic trade balance as a share of GDP

$$\frac{b}{Y}\left(1 - \frac{1+i^*}{\Pi}\right) = \frac{TB}{Y} = v$$

which rearranging provides the household's stock of foreign assets in real terms as

$$b = vY\left(\frac{1}{1 - \frac{1+i^*}{\Pi}} \right)$$

Starting with equation (A7), and substituting equation (A2) we get that steady state real money cash is given by:

$$m^c = \Pi(Y - I - \left(1 - \frac{1+i^*}{\Pi}\right)b)$$

From equation (A11) we get that real money deposits are

$$m^b = \frac{\Pi I - (\theta - 1)m^c}{\theta}$$

Given the definition of real money balances, then real money balances is

$$m = m^b + m^c$$

From the CIA constraint (equation (A2)), steady state consumption is:

$$C = \frac{m^c + \Gamma}{\Pi}$$

From equation (A15) we get that the shadow price associated with household real wealth is in the steady state

$$\Lambda = \frac{\beta}{C \prod}$$

and consequently γ is given by

$$\gamma = w\Lambda(1 - H)$$

III. The log-linearized system of equations is given by

(B1) $0 = -\hat{\pi}_t + \hat{s}_t - \hat{s}_{t-1}$

(B2) $0 = -\hat{\pi}_t - \hat{C}_t + \dfrac{m^c}{C\pi}\hat{m}_t^c + \dfrac{\Gamma}{C\pi}\hat{\Gamma}_t$

(B3) $0 = \hat{w}_t + \hat{\Lambda}_t - \dfrac{H}{1-H}\hat{H}_t$

(B4) $0 = -\hat{Y}_t + \alpha\hat{K}_t + (1-\alpha)\hat{H}_t + \hat{z}_t$

(B5) $0 = \dfrac{I}{K}\hat{I}_t - \hat{K}_{t+1} + (1-\delta)\hat{K}_t$

(B6) $0 = -\hat{w}_t + \hat{Y}_t - \hat{H}_t$

(B7) $$0 = -\hat{b}_{t+1} + \frac{(1+i^*)}{\pi}\hat{s}_t - \frac{(1+i^*)}{\pi}\hat{s}_{t-1} + \frac{(1+i^*)}{\pi}\hat{b}_t - \left(\frac{(1+i^*)b+\Gamma}{b\pi}\right)\hat{\pi}_t + \frac{Y}{b}\hat{Y}_t - \frac{C}{b}\hat{C}_t$$

$$-\frac{I}{b}\hat{I}_t + \frac{\Gamma}{b\pi}\hat{\Gamma}_t + \frac{i^*}{\pi}\hat{i}^*_t$$

(B8) $$0 = -\Delta\hat{M}^c_t + \hat{m}^c_{t+1} + \hat{\pi}_t - \hat{m}^c_t$$

(B9) $$0 = -(m)\hat{m}_{t+1} + (m^b)\hat{m}^b_{t+1} + (m^c)\hat{m}^c_{t+1}$$

(B10) $$0 = -\hat{m}_{t+1} + \hat{m}_t - \hat{\pi}_t + \hat{\theta}_t$$

(B11) $$0 = -\hat{\pi}_t - \hat{I}_t + \frac{m^b}{I\pi}\hat{m}^b_t + \frac{m}{I\pi}(\theta-1)\hat{m}_t + \frac{m\theta}{I\pi}\hat{\theta}_t$$

(B12) $$0 = \underset{t}{E}\left[\beta\upsilon K\hat{K}_{t+2} - \left(\upsilon K + \beta\upsilon K + \frac{\alpha\beta Y}{K}\right)\hat{K}_{t+1} + \upsilon K\hat{K}_t + \beta(1-\delta)(i)\hat{i}_{t+1} - (i)\hat{i}_t\right.$$

$$\left. + \frac{\alpha\beta Y}{K}\hat{Y}_{t+1} + \left(\frac{\alpha\beta Y}{K} + \beta(1-\delta)(1+i))\right)\hat{\Lambda}_{t+1} - \left(\frac{\alpha\beta Y}{K} + \beta(1-\delta)(1+i)\right)\hat{\Lambda}_t\right]$$

(B13) $$0 = \underset{t}{E}\left[-\hat{\Lambda}_t + \frac{i}{1+i}\hat{i}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\pi}_{t+1}\right]$$

(B14) $$0 = \underset{t}{E}\left[-\hat{\Lambda}_t + \hat{\Lambda}_{t+1} + \hat{s}_{t+1} - \hat{s}_t - \hat{\pi}_{t+1} + \frac{i^*}{1+i^*}\hat{i}^*_{t+1}\right]$$

(B15) $$0 = \underset{t}{E}\left[-\Lambda\hat{\Lambda}_t - \frac{\beta}{\pi C}\hat{\pi}_{t+1} - \frac{\beta}{\pi C}\hat{C}_{t+1} + \beta w\Lambda\frac{\xi}{m^c}(\Delta M^c)^2\,\Delta\hat{M}^c_{t+1} - \pi w\Lambda\frac{\xi}{m^c}\Delta M^c\,\Delta\hat{M}^c_t\right]$$

(B16) $$0 = \underset{t}{E}\left[\hat{\pi}_t - \hat{\Gamma}_t - \tau\hat{Y}_t + \hat{s}_t + \hat{g}_t\right]$$

(B17) $$0 = \hat{i}^*_t + \frac{\varphi b}{i^*}\hat{b}_t$$

(B18) $$\hat{\theta}_{t+1} = \rho_\theta\hat{\theta}_t + \varepsilon_{\theta t+1}$$

(B19) $$\hat{g}_{t+1} = \rho_g\hat{g}_t + \varepsilon_{gt+1}$$

(B20) $\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_{zt+1}$

Data Appendix

I extracted quarterly data on remittances from the respective Central Banks, where data was available at this frequency, up to 2010:4.

I also obtained data on real GDP, money (M1 and M2), consumer price index, and trade balance from the International Monetary Fund’s International financial Statistics (IFS), up to 2010:4.

The measures of financial development come from the World Bank (Thorsten and Demirgüç-Kunt (2009)).