# "The Macroeconomic Consequences of Remittances"

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### **Technical Appendix**

### A.1. System of Equations

(A1) 
$$\pi_t = \frac{s_t}{s_{t-1}} \pi^*$$

(A2) 
$$\pi_t C_t = m_t^c + \phi \Gamma_t$$

(A3) 
$$\Lambda_{t} = \beta E_{t} \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} (1+i_{t+1}) \right]$$

(A4) 
$$w_t \Lambda_t = \gamma C_t^{(1-\gamma)(1-\sigma)} (1 - H_t - \Omega_t)^{\gamma(1-\sigma)-1}$$

(A5) 
$$\Lambda_t = \beta E_t \left[ (1 + i_{t+1}^*) \frac{s_{t+1}}{s_t} \frac{\Lambda_{t+1}}{\pi_{t+1}} \right]$$

(A6) 
$$w_t \Lambda_t \xi \frac{\pi_t}{m_t^c} (\Delta M_t^c - \theta) + \Lambda_t = \beta E_t \left[ \frac{(1 - \gamma)C_{t+1}^{-\sigma - \gamma(1 - \sigma)} (1 - H_{t+1} - \Omega_{t+1})^{\gamma(1 - \sigma)}}{\pi_{t+1}} \right] + \beta E_t \left[ w_{t+1} \Lambda_{t+1} \xi \frac{\Delta M_{t+1}^c}{m_{t+1}^c} (\Delta M_{t+1}^c - \theta) \right]$$

(A7) 
$$Y_{t} = e^{z_{t}} K_{t}^{\alpha} H_{t}^{1-\alpha}$$
  
(A8)  $I_{t} = K_{t+1} - (1-\delta)K_{t}$   
(A9)  $w_{t} = (1-\alpha)\frac{Y_{t}}{H_{t}}$ 

(A10) 
$$(1+i_t) + \upsilon(K_{t+1} - K_t) = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)(1+i_{t+1}) + \upsilon(K_{t+2} - K_{t+1}) \right\} \right]$$

(A11) 
$$m_{t+1} = \theta_t \frac{m_t}{\pi_t}$$
  
(A12)  $\pi_t I_t = m_t^b + (\theta_t - 1)m_t + (1 - \phi)\Gamma_t$   
(A13)  $\Gamma_t = \prod_t \left[ \vartheta(Y^{ss})^{\tau} \pi_t s_t Y_t^{-\tau} e^{\vartheta_t} \right]$   
(A14)  $b_{t+1} - \frac{s_t}{s_{t-1}} (1 + i_t^*) \frac{b_t}{\pi_t} = Y_t - C_t - I_t + [1 - (1 + i_t)(1 - \phi)] \frac{\Gamma_t}{\pi_t} - \frac{\upsilon}{2} (K_{t+1} - K_t)^2$   
Define  $\Delta M_t^c = \frac{M_{t+1}^c}{M_t^c}$ , and by previous the definition

(A15) 
$$\Delta M_{t}^{c} = \frac{m_{t+1}^{c} \pi_{t}}{m_{t}^{c}}$$
(A16) 
$$m_{t} = m_{t}^{b} + m_{t}^{c}$$
(A17) 
$$i_{t}^{*} = i^{W} - \varphi b_{t}$$
(A18) 
$$\log(\theta_{t+1}) = (1 - \rho_{\theta})\log(\overline{\theta}) + \rho_{\theta}\log(\theta_{t}) + \varepsilon_{\theta t+1}$$
(A19) 
$$\log(g_{t+1}) = (1 - \rho_{g})\log(\overline{g}) + \rho_{g}\log(g_{t}) + \varepsilon_{gt+1}$$
(A20) 
$$\log(z_{t+1}) = (1 - \rho_{z})\log(\overline{z}) + \rho_{z}\log(z_{t}) + \varepsilon_{zt+1}$$

A.2. The log-linearized system of equations is given by

(B12) 
$$0 = -\hat{\pi}_{t} - I_{t} + \frac{m}{I\pi} \hat{m}_{t}^{b} + \frac{m}{I\pi} (\theta - 1) \hat{m}_{t} + \frac{m}{I} \theta_{t} + \frac{\pi}{I\pi} (1 - \phi)$$
  
(B13) 
$$0 = \prod_{t} \left[ \hat{\pi}_{t} - \hat{\Gamma}_{t} - \tau \hat{Y}_{t} + \hat{s}_{t} + \hat{g}_{t} \right]$$

$$(B14) \quad 0 = -\hat{b}_{t+1} + \frac{(1+i^{*})}{\pi} \hat{s}_{t} - \frac{(1+i^{*})}{\pi} \hat{s}_{t-1} + \frac{(1+i^{*})}{\pi} \hat{b}_{t} + \left(\frac{Y-C-I-b}{b}\right) \hat{\pi}_{t} + \frac{Y}{b} \hat{Y}_{t} - \frac{C}{b} \hat{C}_{t} + \frac{i^{*}}{\pi} \hat{i}_{t}^{*} - \frac{I}{b} \hat{I}_{t} + \left(\frac{(1-(1-\phi)(1+i))\Gamma}{b\pi}\right) \hat{\Gamma}_{t} - \frac{(1-\phi)i\Gamma}{b\pi} \hat{i}_{t}$$

$$(B15) \quad 0 = -\hat{\Delta M}_{t}^{*} + \hat{m}_{t+1}^{c} + \hat{\pi}_{t} - \hat{m}_{t}^{c}$$

$$(B16) \quad 0 = -(m)\hat{m}_{t} + (m^{b})\hat{m}_{t}^{b} + (m^{c})\hat{m}_{t}^{c}$$

$$(B17) \quad 0 = \hat{i}_{t}^{*} + \frac{\phi b}{i^{*}} \hat{b}_{t}$$

$$(B18) \quad \hat{\theta}_{t+1} = \rho_{\theta}\hat{\theta}_{t} + \varepsilon_{\theta+1}$$

$$(B19) \quad \hat{g}_{t+1} = \rho_{g}\hat{g}_{t} + \varepsilon_{gt+1}$$

#### **A3. Steady State Derivations**

In order to provide richer dynamics, we first derive the relative importance of leisure ( $\gamma$ ) and then use that value to derive the hours worked

Given H and solving for  $\gamma$ . Adjustment costs disappear in the steady state, and we assume that in the long-run equilibrium the domestic gross inflation rate is given by the gross money growth rate so that  $\Pi = \theta$ .

From equation (A3) and, after some manipulation, we have that the domestic nominal interest rate in steady state is

$$i = \frac{\prod}{\beta} - 1$$

We look at a steady state in which the domestic and foreign inflation levels are the same, so purchasing power parity implies that the change in the nominal exchange rate is constant.<sup>1</sup> Consequently the uncovered interest rate parity condition – combining equations (A3) and A5) implies that the domestic and the foreign interest rates are equal ( $i = i^*$ ).

<sup>&</sup>lt;sup>1</sup> Note that this assumption sets the steady-state nominal exchange rate to be constant, allowing a different steady-state foreign inflation rate will make the steady-state exchange rate grow at a constant rate.

We can derive the steady state level of remittances from equation (A13) as

 $\Gamma = \mathcal{G} \prod s$ 

From the definition of the change in money balances (equation (A15) we get that

$$\Delta M^{c} = \Pi$$

To find the steady state capital/output ratio (denoted  $\kappa$ ) we get, from the stationarity of equation (A10):

$$1 + i = \beta \left[ \alpha \frac{Y}{K} + (1 - \delta)(1 + i) \right]$$
$$\frac{1 + i}{\beta} - (1 - \delta)(1 + i) = \alpha \frac{Y}{K}$$
$$\kappa = \frac{K}{Y} = \left[ \frac{\alpha \beta}{1 + i - (1 - \delta)(1 + i)\beta} \right]$$

Then from the production function we can solve for the output

$$Y = \kappa^{\frac{\alpha}{1-\alpha}} H$$

which can be used in equation (A9) to solve for the real wage

$$w = (1 - \alpha) \frac{Y}{H}$$

The steady state physical capital stock will be given by  $K = \kappa Y$ , and steady state investment by  $I = \delta K$ .

Since  $TB = Y - C - I + (1 - (1 + i)(1 - \phi))\frac{\Gamma}{\Pi}$  is the adjusted trade balance, we can

divide by *Y* both sides of equation (A14) and use the calibration for v = TB/Y to obtain the long-run real debt-to-GDP ratio, which is equal to the domestic trade balance as a share of GDP

$$\frac{b}{Y}\left(1 - \frac{1 + i^*}{\Pi}\right) = \frac{TB}{Y} = v$$

Such that

$$b = \frac{vY}{\left(1 - \frac{1 + i^*}{\Pi}\right)}$$

Consequently, the steady state consumption level is given by:

$$C = Y + (1 - (1 + i)(1 - \phi))\frac{\Gamma}{\Pi} - I - \left(1 - \frac{1 + i^*}{\Pi}\right)b$$

From the CIA constraint, steady state real money-cash balances are:

$$m^c = \prod C - \phi \Gamma$$

Then using equation (A12) and the definition of money (equation (A16)), the household's steady state real money deposits is

$$m^{b} = \frac{\Pi - (1 - \phi)\Gamma - (\theta - 1)m^{c}}{\theta}$$

Given the definition of real money balances, then its steady state level is:

$$m = m^b + m^c$$

By setting equation (A4) and (A6) equal, we can solve for

$$\gamma = \frac{w\beta}{\Pi\left(\frac{C}{1-H} + \frac{w\beta}{\Pi}\right)}$$

From the definition of preferences, and denoting the shadow price associated with household real wealth by  $\Lambda_t = P_t \lambda_t$ , then the marginal utility of wealth in the steady state is

$$\Lambda = \frac{\gamma C^{(1-\gamma)(1-\sigma)} (1-H)^{\gamma(1-\sigma)-1}}{w}$$

Given  $\gamma$  and solving for H. We assume that the domestic gross inflation rate is given by the gross money growth rate so that  $\Pi = \theta$ . Equation (A3) gives the steady state for the domestic nominal interest rate

$$i = \frac{\prod}{\beta} - 1$$

The uncovered interest rate parity condition – equation (A3) equal to equation (A5) –

implies that the domestic and the foreign interest rates are equal  $(i = i^*)$ .

We can derive the steady state level of remittances from equation (A13) as

$$\Gamma = \mathcal{G} \prod s$$

From the definition of the change in money balances (equation (A15) we get that

$$\Delta M^c = \Pi$$

To find the steady state capital/output ratio (denoted  $\kappa$ ) we get, from the stationarity of equation (A10):

$$\kappa \equiv \frac{K}{Y} = \left[\frac{\alpha\beta}{1+i-(1-\delta)(1+i)\beta}\right]$$

Then from the production function we can solve for the output/labor ratio

$$\frac{Y}{H} = \kappa^{\frac{\alpha}{1-\alpha}}$$

which can be used in equation (15) to solve for the real wage

$$w = (1 - \alpha) \frac{Y}{H}$$

By using equations (A4) and (A6) and solving for  $\Lambda w$  we can set them equal and solve for the consumption/output ratio

$$\frac{C}{Y} = \frac{w\beta(1-\gamma)}{\prod \gamma} \left[ \frac{1}{Y} - \kappa^{-\frac{\alpha}{1-\alpha}} \right]$$

We can also solve for the investment/output ratio from equation (A8)

$$\frac{I}{Y} = \delta \kappa$$

The right-hand side of equation (A14) defines the adjusted trade balance,  $TB = Y - C - I + (1 - (1 + i)(1 - \phi))\frac{\Gamma}{\Pi}$ , which when divided by output allows us to use the calibration v = TB/Y. This and the consumption/output and investment/output ratios allows us to find

$$Y = \frac{\left(\left[1 - (1 + i)(1 - \phi)\right] \mathcal{G}s - \frac{w\beta(1 - \gamma)}{\Pi\gamma}\right)}{\left(v - 1 - \frac{w\beta(1 - \gamma)}{\Pi\gamma}\kappa^{-\frac{\alpha}{1 - \alpha}} + \delta\kappa\right)}$$

Then the steady state physical capital stock will be given by  $K = \kappa Y$ , and steady state investment by  $I = \delta K$ . From the production function we can solve for H

$$H = \frac{Y}{\kappa^{\frac{\alpha}{1-\alpha}}}$$

The steady state stock of foreign assets in real terms is derived from the balance of payments equilibrium (equation (A14)), so the household's stock of foreign assets in real terms is

$$b = vY \left(\frac{1}{1 - \frac{1 + i^*}{\Pi}}\right)$$

By setting equation (A4) and (A6) equal, we can solve for consumption

$$C = \frac{w\beta(1-\gamma)(1-H)}{\Pi\gamma}$$

From the CIA constraint, steady state real money-cash balances are:

$$m^c = \prod C - \phi \Gamma$$

Then using equation (A12) and the definition of money (equation (A16)), the household's steady state real money deposits is

$$m^{b} = \frac{\Pi \Pi - (1 - \phi)\Gamma - (\theta - 1)m^{c}}{\theta}$$

Given the definition of real money balances, then its steady state level is:

$$m = m^b + m^c$$

Back in equation (A4), we can solve for the marginal utility of wealth in the steady state as

$$\Lambda = \frac{\gamma C^{(1-\gamma)(1-\sigma)} (1-H)^{\gamma(1-\sigma)-1}}{w}$$

## A.4. Monetary and Technology Shocks



Figure A1: Dynamic response to a 1% monetary shock Remittances 5% of GDP and 90% going towards Consumption



Figure A2: Dynamic response to a 1% technological shock Remittances 5% of GDP and 90% going towards Consumption