Appendix for “Remittances, Monetary Policy, and Partial Sterilization”, in *Southern Economic Journal*

I. System of Equations

(A1) \[ \pi_t = \frac{s_t}{s_{t-1}} \pi^* \]

(A2) \[ \pi_t C_t = m^*_t + \Gamma_t \]

(A3) \[ \frac{\gamma}{1 - H_t - \Omega_t} = w_t \Lambda_t \]

(A4) \[ Y_t = e^{\gamma_t} K_t^\alpha H_t^{1-\alpha} \]

(A5) \[ I_t = K_{t+1} - (1 - \delta)K_t \]

(A6) \[ w_t = (1 - \alpha) \frac{Y_t}{H_t} \]

(A7) \[ b_{t+1} - \frac{s_t}{s_{t-1}} (1 + i^*_t) \frac{b_t}{\pi_t} = Y_t - C_t - I_t - \frac{\nu}{2} (K_{t+1} - K_t)^2 + \frac{\Gamma_t}{\pi_t} \]

Define \[ \Delta M^c_t \equiv \frac{M^c_{t+1}}{M^c_t} \], and by previous the definition

(A8) \[ \Delta M^c_t = \frac{M^c_{t+1} \pi_t}{m^c_t} \]

(A9) \[ m^c_{t+1} = m^b_{t+1} + m^c_{t+1} \]

(A10) \[ m_{t+1} = \theta_t \frac{m_t}{\pi_t} \]

(A11) \[ \pi_t I_t = m^*_t + (\theta_t - 1)m_t \]
(A12) \( (1 + i_t) + \nu(K_{t+1} - K_t) = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + i_{t+1}) + \nu(K_{t+2} - K_{t+1}) \right\} \right] \)

(A13) \( \Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} (1 + i_{t+1}) \right] \)

(A14) \( \Lambda_t = \beta E_t \left[ (1 + \delta_s^*) \frac{s_{t+1}}{S_t} \frac{\Lambda_{t+1}}{\pi_{t+1}} \right] \)

(A15) \( w_t \Lambda_t \hat{\pi} \frac{\Lambda_{t+1}^s}{m_t^c} (\Delta M_t^c - \theta) + \Lambda_t = \beta E_t \left[ \frac{1}{C_{t+1} \pi_{t+1}} \frac{\Lambda_{t+1}}{\pi_{t+1}} \right] + \beta E_t \left[ w_{t+1} \Lambda_{t+1} \hat{\pi} \frac{\Delta M_t^c}{m_{t+1}^c} (\Delta M_t^c - \theta) \right] \)

(A16) \( \Gamma_t = E_t \left[ \theta(Y^n)^r \pi_t \sigma_t Y_t^{-r} e^{\delta_t} \right] \)

(A17) \( \delta_s^* = \delta_w - \phi \theta_t \)

(A18) \( \log(\theta_{t+1}) = (1 - \rho_\theta) \log(\bar{\theta}) + \rho_\theta \log(\theta_t) + \epsilon_{\theta_{t+1}} \)

(A19) \( \log(g_{t+1}) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_t) + \epsilon_{g_{t+1}} \)

(A20) \( \log(z_{t+1}) = (1 - \rho_z) \log(\bar{z}) + \rho_z \log(z_t) + \epsilon_{z_{t+1}} \)

II. Steady States

In the long-run equilibrium we assume the domestic gross inflation rate is given by the gross money growth rate so that \( \Pi = \theta \). From (A13) we have that the domestic nominal interest rate in steady state is

\[ i = \frac{\Pi}{\beta} - 1 \]

We can derive the steady state level of remittances from equation (A16) as

\[ \Gamma = \beta \Pi s \]

To find the steady state capital/output ratio (denoted \( \kappa \)) we use equation (A12) to get:
\[
\kappa \equiv \frac{K}{Y} = \left[ \frac{\alpha \beta}{1 + i - (1 - \delta)(1 + i)\beta} \right]
\]

Then from the production function we can solve for the output/labor ratio

\[
\frac{Y}{H} = \kappa^{\frac{1}{1 - \alpha}}
\]

such that \( Y = H^{\frac{\alpha}{1 - \alpha}} \), \( K = \frac{Y^{\frac{1}{1 - \alpha}}}{H^{\frac{1}{1 - \alpha}}} \), and \( I = \delta K \).

From equation (A6) we solve for the real wage

\[
w = (1 - \alpha) \frac{Y}{H}
\]

Setting equation (A13) equal to equation (A14) we get that \( i = i^* \).

The adjusted trade balance is given by \( \Pi T B = Y - C - I + \Gamma \). Using the calibration for \( \nu = TB/Y \), we obtain the long-run real debt-to-GDP ratio that is equal to the domestic trade balance as a share of GDP

\[
\frac{b}{Y} \left( 1 - \frac{1 + i^*}{\Pi} \right) = \frac{TB}{Y} = \nu
\]

which rearranging provides the household’s stock of foreign assets in real terms as

\[
b = \nu \left( 1 - \frac{1 + i^*}{\Pi} \right)
\]

Starting with equation (A7), and substituting equation (A2) we get that steady state real money cash is given by:

\[
m^c = \Pi(Y - I - \left( 1 - \frac{1 + i^*}{\Pi} \right)b)
\]
From equation (A11) we get that real money deposits are

\[ m^b = \frac{\Pi I - (\theta - 1)m^c}{\theta} \]

Given the definition of real money balances, then real money balances is

\[ m = m^b + m^c \]

From the CIA constraint (equation (A2)), steady state consumption is:

\[ C = \frac{m^c + \Gamma}{\Pi} \]

From equation (A15) we get that the shadow price associated with household real wealth is in the steady state

\[ \Lambda = \frac{\beta}{C\Pi} \]

and consequently \( \gamma \) is given by

\[ \gamma = w\Lambda(1 - H) \]

III. The log-linearized system of equations is given by

(B1) \[ 0 = -\hat{\pi}_t + \hat{s}_t - \hat{s}_{t-1} \]

(B2) \[ 0 = -\hat{\pi}_t - \hat{C}_t + \frac{m^c}{C\pi} \hat{m}_t^c + \frac{\Gamma}{C\pi} \hat{\bar{C}}_t \]

(B3) \[ 0 = \hat{w}_t + \hat{A}_t - \frac{H}{1 - H} \hat{\bar{H}}_t \]

(B4) \[ 0 = -\hat{Y}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{\bar{H}}_t + \hat{z}_t \]

(B5) \[ 0 = \frac{\hat{I}}{K} - \hat{K}_{t+1} + (1 - \delta) \hat{\bar{K}}_t \]

(B6) \[ 0 = -\hat{w}_t + \hat{Y}_t - \hat{\bar{H}}_t \]
(B7) \( 0 = -\hat{b}_{t+1} + \frac{(1+i^*)}{\pi} \hat{s}_t - \frac{(1+i^*)}{\pi} \hat{s}_{t-1} + \frac{(1+i^*)}{\pi} \hat{b}_t - \left( \frac{(1+i^*)b + \Gamma}{b\pi} \right) \hat{b}_t + \frac{Y}{b} \hat{Y}_t - \frac{C}{b} \hat{C}_t \)

\[ -\frac{I}{b} \hat{I}_t + \frac{\Gamma}{b\pi} \hat{I}_t + \frac{i^*}{\pi} \hat{i}_t^* \]

(B8) \( 0 = -\Delta M^c_t + \dot{\hat{m}}^c_{t+1} + \hat{\pi}_t - \ddot{m}^c_t \)

(B9) \( 0 = -(m^\hat{b}) \dot{\hat{m}}^b_{t+1} + (m^\hat{b}) \dot{m}^b_{t+1} + (m^\hat{b}) \ddot{m}^b_{t+1} \)

(B10) \( 0 = -\ddot{\hat{m}}_{t+1} + \dot{\hat{m}}_t - \ddot{\pi}_t + \ddot{\theta}_t \)

(B11) \( 0 = -\ddot{\pi}_t - \dot{\hat{n}}_t + \frac{m^b}{I\pi} \ddot{\hat{m}}^b_t + \frac{m}{I\pi} (\theta - 1) \ddot{\hat{m}}_t + \frac{m\theta}{I\pi} \ddot{\theta}_t \)

(B12) \( 0 = \mathbf{E}_t \left[ \beta \nu K \hat{\mathbf{K}}_{t+2} - (\nu K + \beta \nu K + \frac{\alpha \beta Y}{K}) \hat{\mathbf{K}}_{t+1} + \nu K \hat{\mathbf{K}}_t + \beta (1 - \delta)(i) \hat{i}_{t+1} - (i) \dot{i}_t \right. \)

\[ + \frac{\alpha \beta Y}{K} \hat{Y}_{t+1} + \left( \frac{\alpha \beta Y}{K} + \beta (1 - \delta)(1 + i) \right) \hat{\lambda}_{t+1} - \left( \frac{\alpha \beta Y}{K} + \beta (1 - \delta)(1 + i) \right) \hat{\lambda}_t \]

(B13) \( 0 = \mathbf{E}_t \left[ -\ddot{\hat{\lambda}}_t + \frac{i^*}{1+i^*} \dot{i}_{t+1} + \hat{\lambda}_{t+1} - \ddot{\pi}_{t+1} \right] \]

(B14) \( 0 = \mathbf{E}_t \left[ -\ddot{\hat{\lambda}}_t + \ddot{\hat{\lambda}}_{t+1} + \ddot{s}_{t+1} - \ddot{s}_t - \ddot{\pi}_{t+1} + \frac{i^*}{1+i^*} \dot{i}_{t+1}^* \right] \]

(B15) \( 0 = \mathbf{E}_t \left[ -\ddot{\hat{\lambda}}_t - \frac{\beta}{\pi C} \ddot{\hat{\pi}}_{t+1} - \frac{\beta}{\pi C} \ddot{\hat{\pi}}_t + \beta \pi \Lambda - \frac{\xi}{m^c} (\Delta M^c)^2 \Delta M^c_{t+1} - \pi \omega \Lambda - \frac{\xi}{m^c} \Delta M^c \Delta M^c_t \right] \]

(B16) \( 0 = \mathbf{E}_t \left[ \ddot{\hat{\pi}}_t - \ddot{\hat{\pi}}_t - \tau \ddot{\hat{\pi}}_t + \ddot{s}_t + \ddot{\hat{g}}_t \right] \]

(B17) \( 0 = \ddot{i}_t + \frac{\varphi^b}{i^*} \hat{b}_t \)

(B18) \( \ddot{\hat{\theta}}_{t+1} = \rho_\theta \ddot{\hat{\theta}}_t + \epsilon_{\theta + t} \)

(B19) \( \ddot{\hat{g}}_{t+1} = \rho_\xi \ddot{\hat{g}}_t + \epsilon_{\xi + t} \)
\[(B20) \quad \hat{z}_{t+1} = \rho \hat{z}_t + \varepsilon_{zt+1}\]

**Data Appendix**

I extracted quarterly data on remittances from the respective Central Banks, where data was available at this frequency, up to 2010:4.

I also obtained data on real GDP, money (M1 and M2), consumer price index, and trade balance from the International Monetary Fund’s International financial Statistics (IFS), up to 2010:4.

The measures of financial development come from the World Bank (Thorsten and Demirgüç-Kunt (2009)).